A New Entropy Based Fuzzy Clustering Algorithm for Volumetric Noisy Brain MR Image Segmentation

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***Abstract*—In this paper we have proposed a new entropy based fuzzy clustering algorithm for segmentation of volumetric noisy brain MR image data. The algorithm utilizes intensity distribution from spatial cubic local neighborhood characterizing a possibility measure that defines likeliness of a voxel under consideration to belong into a cluster or region. This is realized by judiciously defining a Gaussian density function. We then normalized these likeliness measures to use them as an alternative membership function. In addition to the fuzzy membership function, this normalized likeliness measure is also incorporated into the objective function using a regularizing parameter that resolves the trade-off between these two terms. Finally, a fuzzy entropy defined by Shannon's function using the normalized likeliness measures is introduced that defines the vagueness and ambiguity uncertainty while classifying a voxel into its possible cluster. Therefore, the cluster prototypes of the proposed algorithm utilize the fuzzy membership functions, likeliness measures and fuzzy entropy. To validate the algorithm, we have performed both qualitative and quantitative analysis on noisy simulated and clinical brain MR image volumes. Its results are found to be superior while comparing with some of the state-of-the-art algorithms.**

I. Introduction

Brain MR image segmentation has gained popularity in recent past due to its nature of treatment and consistent search for near perfect clinical diagnosis and analysis. There are numerous different types of methods for MR image segmentation have been proposed. However, among them, the fuzzy logic based clustering algorithms are studied more as it deals with uncertainties that arise while classifying pixels into possible brain soft tissue regions. But, conventional fuzzy clustering algorithm like fuzzy c-means (FCM) algorithm does not consider spatial information, thereby limiting its performance, especially when the brain MR images are corrupted by high noise and intensity inhomogeneity (IIH). Due to this shortcoming many authors have modified the FCM algorithm to make it robust to noise and IIH. Chung *et al.* [1] used spatial information in FCM algorithm for image segmentation. It uses spatial information in the form of summation of membership functions from a immediate neighborhood of the pixel under consideration. Later, some modified FCM algorithms are proposed using conditional variables [2], spatial and membership functions [3]. Pal *et al.* [4] proposed another improvement using possibilistic measures and membership functions. Kahali *et al.* [5] proposed a two–stage fuzzy multi-objective framework for 3D brain MR image segmentation. It uses global and local membership functions along with 3D spatial neighborhood information.

The works described above do not use entropy. However, some authors proposed fuzzy clustering algorithm by using entropy for machine learning data set [6]. It automatically identifies the number and locations of initial cluster centers by minimizing entropy. Kannan *et al.* [7] used effective quadratic entropy by the combination of mean distance, kernel distance and regularization function and quadratic term for time series data. Zarinbal *et al.* [8] proposed relative entropy based FCM algorithm for noisy data. Askari *et al.* [9] combined entropy and possibilistic measure to propose possibilistic fuzzy c-means (PFCM) algorithm for clustering of noisy data.

In this paper we have proposed a fuzzy entropy based clustering algorithm for segmentation of volumetric noisy brain MR image data. For each voxel under consideration, we have defined likeliness measures corresponding to all clusters from its cubic local neighborhood region. These measures are finally used to introduce fuzzy entropy that defines the vagueness and information uncertainty while identifying the possible cluster or class. The fuzzy membership function, likeliness measures and fuzzy entropy are judiciously incorporated into the objective function. The simulation results on simulated and clinical noisy 3D brain MR image data show that the algorithm is superior to some of the state-of-the-art algorithms.

II. Proposed Entropy Based Fuzzy Clustering Algorithm

As brain MR images are contaminated by noise and intensity inhomogeneity (IIH) [10], achieving accurate segmentation results quite difficult and challenging task. The task even become near impossible and prone to error while classifying the voxels at the tissue boundaries since information uncertainty attains to its maximum value. Therefore, better representation and utilization of this information uncertainty into a fuzzy clustering algorithm will lead to a possible solution of the problem. To realize this we have utilized intensity distribution in a spatial cubic local neighborhood to measure a possibility factor that defines likeliness of the voxel under consideration to belong into a cluster or region. To mitigate the affects of noise and IIH, this is characterized by judiciously defining a Gaussian density function. Afterwards these likeliness measures are normalized so as to consider them as an alternative membership function. The fuzzy membership function and this normalized likeliness measure are incorporated into the objective function using a regularizing parameter. Additionally, normalized likeliness measure based fuzzy entropy defined by Shannon's function is introduced to represent the vagueness and ambiguity uncertainty while classifying a voxel into its possible cluster. Therefore, this framework makes the cluster prototypes to utilize the fuzzy membership functions, likeliness measures and fuzzy entropy. Further, to mitigate the affects of noise and IIH, for each voxel x_{zrc} we construct a vector X_{zrc} of 8 features consisting mean intensity values of four diagonals, 3 cross sections and the center with respect its cubic local neighborhood. For a brain MR image volume of size $Z \times M \times N$ ($depth \times height \times width$) with C different soft tissue regions, by considering the above points the objective function of the proposed algorithm can be defined as follows:

$$\begin{aligned} J = &\sum_{i=1}^{C}\sum_{z=1}^{Z}\sum_{r=1}^{M}\sum_{c=1}^{N}[\alpha\mu_{i_{zrc}}^{m}||X_{zrc}-V_i||^2 \\ &+ (1-\alpha)p_{i_{zrc}}^{m}||X_{zrc}-V_i||^2] \\ &- \sum_{z=1}^{Z}\sum_{r=1}^{M}\sum_{c=1}^{N}\sum_{i=1}^{C}[p_{i_{zrc}}ln(p_{i_{zrc}})] \end{aligned} \quad (1)$$

subject to the following constraint,

$$\sum_{i=1}^{C}\mu_{i_{zrc}} = 1 \quad (2)$$

where α is the regularizing parameter (> 0.0) to control the trade-off between fuzzy membership functions and likeliness measures, m is the fuzzifier (> 1.0). $\mu_{i_{zrc}}$ is the fuzzy membership function or degree of membership of the voxel x_{zrc} to belong into the i^{th} cluster center V_i, $p_{i_{zrc}}$ is the normalized likeliness measure, and $||*||$ is the Euclidean norm expressing the dissimilarity between any measured data and the center (here voxel and cluster center, respectively).

The likeliness measure that defines a possibility factor of a voxel to belong into a cluster is defined using the intensity distribution of its cubic local neighborhood. This measure is inversely proportional to its distance from the cluster center, similar to the fuzzy membership function. Further, to use them as alternative membership functions, we normalized them so that their summation over all the clusters become 1.0. These are realized by the following equations.

$$p_{i_{zrc}} = \frac{G_{i_{zrc}}}{\sum_{i=1}^{C} G_{i_{zrc}}} \quad (3)$$

$$G_{i_{zrc}} = \frac{e^{\frac{-||X_{zrc}-V_i||^2}{2\sigma_i^2}}}{\sum_{l=1|X_l\in N_{zrc}} e^{\frac{-||X_l-V_i||^2}{2\sigma_i^2}}} \quad (4)$$

The iterative equations for the fuzzy membership function $\mu_{i_{zrc}}$ and cluster center V_i can be found by combining the objective function in (1) and constraint in (2) with the help of Lagrange multipliers and setting zero the corresponding partial derivatives as defined follows:

$$\frac{\partial}{\partial\mu_{i_{zrc}}}(J) = 0 \quad \text{and} \quad \frac{\partial}{\partial V_i}(J) = 0 \quad (5)$$

From (5), we get the following final iterative equations as follows:

$$\mu_{i_{zrc}} = \frac{1}{\left[\frac{d_{i_{zrc}}^2}{\sum_{j=1}^{C} d_{j_{zrc}}^2}\right]^{\frac{1}{m-1}}} \quad (6)$$

$$V_i = \frac{f(A)}{f(B)} \quad (7)$$

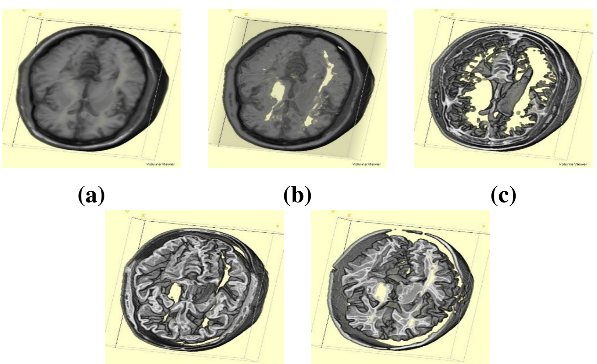
$$\begin{aligned} f(A) = \sum_{z=1}^{Z}\sum_{r=1}^{M}\sum_{c=1}^{N}\Bigg[&2\alpha\mu_{i_{zrc}}^{m}X_{zrc} + 2(1-\alpha)p_{i_{zrc}}^{m}X_{zrc} \\ &+ \frac{(1-\alpha)X_{zrc}d_{i_{zrc}}^2 G_{i_{zrc}}}{\sigma_i^2(\sum_{l=1}^{C} G_{l_{zrc}})} - \frac{(1-\alpha)\overline{X_l}d_{i_{zrc}}^2 G_{i_{zrc}}}{\sigma_i^2(\sum_{l=1}^{C} G_{l_{zrc}})} \\ &+ \frac{X_{zrc}G_{i_{zrc}}ln(p_{i_{zrc}})}{\sigma_i^2(\sum_{l=1}^{C} G_{l_{zrc}})} - \frac{\overline{X_l}G_{i_{zrc}}ln(p_{i_{zrc}})}{\sigma_i^2(\sum_{l=1}^{C} G_{l_{zrc}})} \\ &+ \frac{X_{zrc}G_{i_{zrc}}}{\sigma_i^2(\sum_{l=1}^{C} G_{l_{zrc}})} - \frac{\overline{X_l}G_{i_{zrc}}}{\sigma_i^2(\sum_{l=1}^{C} G_{l_{zrc}})}\Bigg] \end{aligned} \quad (8)$$

$$\begin{aligned} f(B) = \sum_{z=1}^{Z}\sum_{r=1}^{M}\sum_{c=1}^{N}\Bigg[&2\alpha\mu_{i_{zrc}}^{m} + 2(1-\alpha)p_{i_{zrc}}^{m} \\ &+ \frac{(1-\alpha)d_{i_{zrc}}^2 G_{i_{zrc}}}{\sigma_i^2(\sum_{l=1}^{C} G_{l_{zrc}})} - \frac{(1-\alpha)N_b d_{i_{zrc}}^2 G_{i_{zrc}}}{\sigma_i^2(\sum_{l=1}^{C} G_{l_{zrc}})} \\ &+ \frac{G_{i_{zrc}}ln(p_{i_{zrc}})}{\sigma_i^2(\sum_{l=1}^{C} G_{l_{zrc}})} - \frac{N_b G_{i_{zrc}}ln(p_{i_{zrc}})}{\sigma_i^2(\sum_{l=1}^{C} G_{l_{zrc}})} \\ &+ \frac{G_{i_{zrc}}}{\sigma_i^2(\sum_{l=1}^{C} G_{l_{zrc}})} - \frac{N_b G_{i_{zrc}}}{\sigma_i^2(\sum_{l=1}^{C} G_{l_{zrc}})}\Bigg] \end{aligned} \quad (9)$$

where $d_{i_{zrc}}^2 = ||X_{zrc} - V_i||^2$, $\overline{X_l} = \sum_{l=1|X_l\in N_{zrc}}^{N_{zrc}} X_l$ and N_b is the size of local neighborhood of the voxel under consideration.

III. Experimental Results and Discussion

The performance of the proposed method with $m = 2.75$, $\alpha = 0.9$ and $7 \times 7 \times 7$ neighborhood is first evaluated on the BrainWeb [11] volumetric brain MR dataset having high noise and IIH, and later on clinical brain MR volume data both in quantitatively and qualitatively. The BrainWeb and clinical data contain six and four volumes, respectively. The image volumes are segmented into four main soft tissue regions that constitute the brain *i. e.* cerebrospinal fluid (CSF), gray matter (GM), white matter (WM) and background. The performance of the proposed method is also compared with the FCM [12], [13], FGFCM [14], sFCM [1], ASIFC [15], PFCM [4], 2sFMoF [5] methods. Fig. 1 shows the segmentation results of the proposed method on a T1-weighted MR image volume having 9% noise, 40% inhomogeneity. The results prove that it can effectively segments the soft tissue regions. In Table I we have presented a comparative study in terms of segmentation accuracy (SA) [2] between the proposed and earlier methods. As we can see that the proposed algorithm is superior and robust with increasing amount of noise and IIH.



(a) **(b)** **(c)** **(d)** **(e)**

Fig. 1. Segmentation results. (a): MR image volume, (b): Segmented image volume, (c): CSF volume, (d): GM volume and (e): WM volume.

TABLE I

A Comparative Tabulation in Terms of Segmentation Accuracy (SA) for BrainWeb Data

Image volumes (Noise% - IIH%)	Regions	Segmentation Accuracy (SA)						
		FCM	FGFCM	sFCM	ASIFC	PFCM	2sFMoF	Proposed Method
5 – 20	CSF	0.881	0.861	0.907	0.911	0.907	0.918	0.904
	GM	0.834	0.828	0.916	0.919	0.879	0.923	0.905
	WM	0.848	0.941	0.938	0.946	0.959	0.968	0.972
5 – 40	CSF	0.837	0.832	0.861	0.867	0.875	0.908	0.901
	GM	0.825	0.821	0.909	0.911	0.837	0.919	0.888
	WM	0.840	0.916	0.926	0.933	0.925	0.962	0.945
7 – 20	CSF	0.819	0.816	0.852	0.859	0.836	0.901	0.895
	GM	0.818	0.801	0.902	0.907	0.815	0.911	0.892
	WM	0.829	0.909	0.912	0.918	0.949	0.954	0.967
7 – 40	CSF	0.807	0.795	0.849	0.852	0.817	0.898	0.891
	GM	0.782	0.792	0.895	0.989	0.772	0.902	0.877
	WM	0.795	0.906	0.903	0.908	0.926	0.947	0.940
9 – 20	CSF	0.753	0.739	0.827	0.836	0.777	0.880	0.874
	GM	0.755	0.736	0.871	0.875	0.762	0.894	0.877
	WM	0.781	0.873	0.897	0.901	0.932	0.942	0.958
9 – 40	CSF	0.742	0.731	0.824	0.829	0.753	0.876	0.871
	GM	0.742	0.725	0.862	0.868	0.737	0.879	0.861
	WM	0.765	0.876	0.873	0.880	0.914	0.923	0.931

Comparative graphical presentations of different algorithms in terms of partition coefficient (V_{pc}) [2], partition entropy (V_{pe}) [2] and Dice similarity coefficient (DSC) or Dice coefficient [2] are shown in Fig. 2, Fig. 3 and Fig. 4, respectively. We can again observe that the proposed algorithm yields superior results in all the cases.

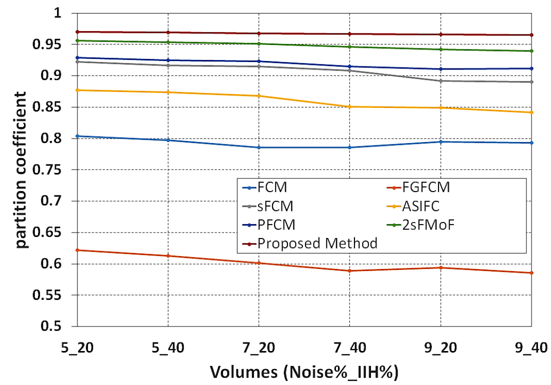


Fig. 2. Segmentation results of different algorithms in terms of partition coefficient

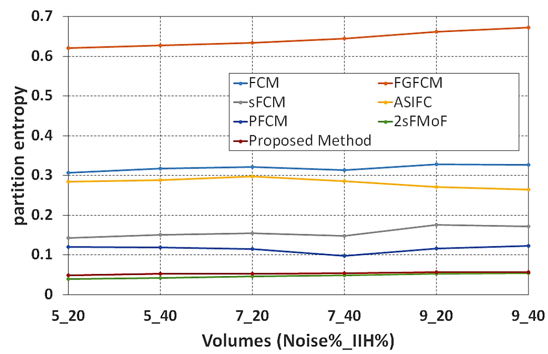


Fig. 3. Segmentation results of different algorithms in terms of partition entropy

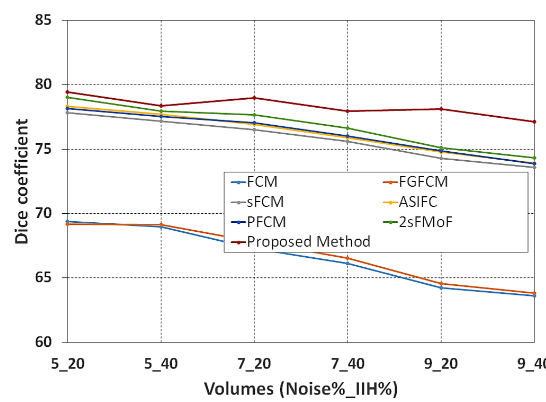


Fig. 4. Segmentation results of different algorithms in terms of Dice coefficient

Fig. 5 presents the qualitative segmentation results on a clinical brain MR image volume, which is again quite effectively segmented. Another comparison between the different algorithms in terms of partition coefficient (V_{pc}) [2] and partition entropy (V_{pe}) [2] are shown in Table II. The results again demonstrate that the proposed algorithm yields superior results in all the cases.

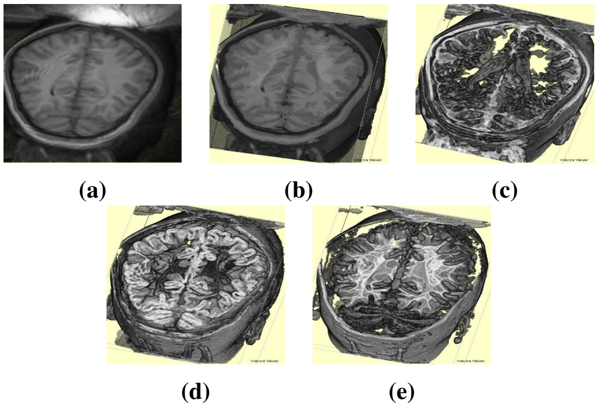


Fig. 5. Segmentation results on clinical brain MR image volumes. (a): Input image volume, (b): Segmented image volume, (c): CSF volume, (d): GM volume and (e): WM volume

TABLE II

COMPARATIVE STUDY IN TERMS OF V_{pc} AND V_{pe} ON CLINICAL BRAIN MR IMAGE VOLUMES

Image volume	Method	V_{pc}	V_{pe}
Clinical data 1 (Male)	FCM	0.705	0.558
	FGFCM	0.815	0.329
	sFCM	0.886	0.294
	ASIFC	0.911	0.206
	PFCM	0.924	0.137
	2sFMoF	0.917	0.032
	Proposed Method	0.956	0.074
Clinical data 2 (Female)	FCM	0.791	0.253
	FGFCM	0.811	0.159
	sFCM	0.835	0.074
	ASIFC	0.897	0.053
	PFCM	0.905	0.087
	2sFMoF	0.953	0.087
	Proposed Method	0.978	0.038
Clinical data 3 (Female)	FCM	0.741	0.507
	FGFCM	0.852	0.287
	sFCM	0.883	0.228
	ASIFC	0.912	0.196
	PFCM	0.921	0.129
	2sFMoF	0.925	0.093
	Proposed Method	0.983	0.028
Clinical data 4 (Female)	FCM	0.870	0.273
	FGFCM	0.893	0.193
	sFCM	0.907	0.178
	ASIFC	0.922	0.143
	PFCM	0.935	0.112
	2sFMoF	0.932	0.109
	Proposed Method	0.974	0.044

IV. CONCLUSION

In this paper we have presented a fuzzy entropy based clustering algorithm for volumetric brain MR image segmentation with high noise and intensity inhomogeneity. Fuzzy entropy is defined using normalized likeliness measures, which is characterized by a Gaussian density function by incorporating local intensity distribution. The likeliness measure is inversely proportional to the distance from a cluster center. This formulation makes the cluster centers to exploits the information regarding fuzzy membership functions, likeliness measures and fuzzy entropy. This proposed algorithm is extensively studied both in qualitatively and quantitatively on simulated and clinical brain MR image volumes with high noise and IIH. The performance of it is superior to some of the state-of-the-art algorithms.

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REFERENCES

[1] K.-S. Chuang, H.-L. Tzeng, S. Chen, J. Wu, and T.-J. Chen, "Fuzzy c-means clustering with spatial information for image segmentation," *computerized medical imaging and graphics*, vol. 30, no. 1, pp. 9–15, 2006.

[2] S. K. Adhikari, J. K. Sing, D. K. Basu, and M. Nasipuri, "Conditional spatial fuzzy c-means clustering algorithm for segmentation of MRI images," *Applied soft computing*, vol. 34, pp. 758–769, 2015.

[3] J. K. Sing, S. K. Adhikari, and D. K. Basu, "A modified fuzzy c-means algorithm using scale control spatial information for MRI image segmentation in the presence of noise," *Journal of Chemometrics*, vol. 29, pp. 492–505, 2015.

[4] N. R. Pal, K. Pal, J. M. Keller, and J. C. Bezdek, "A possibilistic fuzzy c-means clustering algorithm," *IEEE transactions on fuzzy systems*, vol. 13, no. 4, pp. 517–530, 2005.

[5] S. Kahali, S. K. Adhikari, and J. K. Sing, "A two-stage fuzzy multi-objective framework for segmentation of 3D MRI brain image data," *Applied Soft Computing*, vol. 60, pp. 312–327, 2017.

[6] J. Yao, M. Dash, S. Tan, and H. Liu, "Entropy-based fuzzy clustering and fuzzy modeling," *Fuzzy sets and Systems*, vol. 113, no. 3, pp. 381–388, 2000.

[7] S. Kannan, S. Ramathilagam, and P.-C. Chung, "Effective fuzzy c-means clustering algorithms for data clustering problems," *Expert Systems with Applications*, vol. 39, no. 7, pp. 6292–6300, 2012.

[8] M. Zarinbal, M. F. Zarandi, and I. Turksen, "Relative entropy fuzzy c-means clustering," *Information Sciences*, vol. 260, pp. 74–97, 2014.

[9] S. Askari, N. Montazerin, M. F. Zarandi, and E. Hakimi, "Generalized entropy based possibilistic fuzzy c-means for clustering noisy data and its convergence proof," *Neurocomputing*, vol. 219, pp. 186–202, 2017.

[10] S. D. Olabarriaga and A. W. Smeulders, "Interaction in the segmentation of medical images: A survey," *Medical image analysis*, vol. 5, no. 2, pp. 127–142, 2001.

[11] "Brainweb: Simulated brain database," [Online; accessed 17-September-2018]. [Online]. Available: http://www.bic.mni.mcgill.ca/brainweb/

[12] J. C. Dunn, "A fuzzy relative of the ISODATA process and its use in detecting compact well-separated clusters," *Journal of Cybernetics*, vol. 3, no. 3, pp. 32–57, 1973.

[13] J. C. Bezdek, "Pattern recognition with fuzzy objective function algorithms," 1981.

[14] W. Cai, S. Chen, and D. Zhang, "Fast and robust fuzzy c-means clustering algorithms incorporating local information for image segmentation," *Pattern recognition*, vol. 40, no. 3, pp. 825–838, 2007.

[15] Z. Wang, Q. Song, Y. C. Soh, and K. Sim, "An adaptive spatial information-theoretic fuzzy clustering algorithm for image segmentation," *Computer Vision and Image Understanding*, vol. 117, no. 10, pp. 1412–1420, 2013.